

## REFLECTIONLESS ABSORPTION OF ELECTROMAGNETIC RADIATION IN ITS INCIDENCE ON THE TWO-LAYER SYSTEM DIELECTRIC–METAL AT AN ANGLE

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*A study is made of the conditions of occurrence of the total reflectionless absorption of electromagnetic radiation in its incidence, at an angle, on a layer of an absorbing dielectric applied to a metal substrate. The equations of the relationship between the selective values of the angle of incidence of the wave and the thickness of the coating layer and its dielectric properties for which we have the effect of total absorption of the incident wave with a different type of its polarization are obtained. The possibility of experimental observation of the indicated effect is considered.*

In [2], it has been substantiated theoretically that in normal incidence of plane-parallel electromagnetic radiation on the two-layer system dielectric–metal, we have the phenomenon of total or reflectionless absorption of the radiation with a prescribed frequency for certain selective values of the dielectric-layer thickness and of the dielectric properties of the coating material. This phenomenon occurs at the so-called zero minima of the oscillating and decaying dependence of the modulus of the reflection coefficient of the wave on the thickness of the coating layer. The equations describing the existence conditions of this phenomenon can be obtained in the general case from the condition of equality of the input resistance  $Z_{in}$  of the two-layer system and the wave resistance  $Z_0$  of vacuum.

Experimental confirmations of the existence of the reflectionless absorption of waves in a substance were obtained in [2] in the microwave range with the example of investigation of the characteristics of the reflection of microwave radiation by binary solutions of polar liquids in nonpolar solvents. At a prescribed frequency of the incident radiation, the absorption of the wave in the solutions was total for the strictly specified (inherent in this solution) thickness of the layer and concentrations of the polar component of the solution.

We believe that this phenomenon can also occur in the cases where electromagnetic radiation is incident on a plane two-layer system at an angle to its surface. To solve this problem we consider the incidence of a plane wave at an angle  $\alpha_0$  to the surface of a plane layer of a dielectric having a complex value of the permittivity  $\epsilon$  and applied on an ideal metal substrate. With allowance for the position of the vector of electric polarization  $\mathbf{E}$  of the wave in relation to the plane of its incidence, we will differentiate the cases of the reflection of a parallel polarized (PP) wave and a transverse polarized (TP) wave respectively when the vector  $\mathbf{E}$  is parallel or perpendicular to the plane of incidence of the wave.

In incidence of the plane wave on the system dielectric–metal at an angle, the propagation constant of the wave  $\gamma$  in the substance of the coating is equal to

$$\gamma = \gamma_0 \frac{\cos \alpha}{\cos \alpha_0}, \quad (1)$$

where  $\gamma_0 = i2\pi/\lambda$  and  $\cos \alpha = \sqrt{1 - p/\epsilon}$ ,  $p = \sin^2 \alpha_0$  [3].

Depending on the type of polarization of the incident wave, the modulus of the coefficient of its reflection  $\rho$  from the plane two-layer system in question is equal to

$$\rho = \begin{cases} \frac{Z_{\text{in}} \cos \alpha_0 - Z_0 \cos \alpha}{Z_{\text{in}} \cos \alpha_0 + Z_0 \cos \alpha} & \text{in reflection of the TP wave,} \\ \frac{Z_0 \cos \alpha_0 - Z_{\text{in}} \cos \alpha}{Z_0 \cos \alpha_0 + Z_{\text{in}} \cos \alpha} & \text{in reflection of the PP wave,} \end{cases} \quad (2)$$

where  $Z_{\text{in}} = Z \tanh \gamma l$ .

The reflectionless absorption of the wave in the two-layer system in question can occur at the point of minimum of  $\rho$  as a function of  $l$  and when the condition  $\rho = 0$  is fulfilled at this point. Since  $Z = Z_0/\sqrt{\varepsilon}$ , with account for expression (1) we have

$$\tanh \frac{2\pi l}{\lambda} \sqrt{\varepsilon - p} = \begin{cases} \frac{Z_0 \cos \alpha}{Z \cos \alpha_0} = \frac{\sqrt{\varepsilon - p}}{\sqrt{1 - p}} & \text{for the TP wave,} \\ \frac{Z_0 \cos \alpha_0}{Z \cos \alpha} = \frac{\varepsilon \sqrt{1 - p}}{\sqrt{\varepsilon - p}} & \text{for the PP wave.} \end{cases} \quad (3)$$

The quantity  $\varepsilon$  entering into Eq. (3) is equal to  $\varepsilon' - i\varepsilon''$ , and  $\varepsilon'$  and  $\varepsilon''$  are related to the refractive index  $n$  and the dielectric loss factor  $y$  of this substance by the known equations

$$\varepsilon' = n^2 (1 - y^2), \quad \varepsilon'' = 2n^2 y, \quad (4)$$

where  $n = \frac{\lambda}{\lambda_d}$ ,  $y = \tan \frac{\delta}{2}$ , and  $\delta = \arctan \frac{\varepsilon''}{\varepsilon'}$ .

For the convenience of further consideration we introduce the notation

$$\varepsilon_1 = \frac{\varepsilon' - p}{1 - p}, \quad \varepsilon_2 = \frac{\varepsilon''}{1 - p}, \quad (5)$$

in which  $\varepsilon_1$  and  $\varepsilon_2$ , by analogy with expressions (4), can be represented in the form

$$\varepsilon_1 = n_1^2 (1 - y_1^2), \quad \varepsilon_2 = 2n_1^2 y_1, \quad (6)$$

where  $n_1 = \lambda_1/\lambda_{1d}$  and  $y_1 = \tan \delta_1$ ;  $\delta_1 = \arctan (\varepsilon_2/\varepsilon_1)$ ,  $\lambda_1 = \lambda \sqrt{1 - p}$ .

Employing this notation in Eqs. (3), upon transformations we obtain

$$\tanh (2\pi x y_1 + i2\pi x) = N (1 - iY), \quad (7)$$

where  $x = l/\lambda_{1d}$ .

Depending on the type of polarization of the wave,  $N$  and  $Y$  entering into Eq. (7) have the form

$$N = n_1, \quad Y = y_1 \quad \text{for the TP wave;} \\ N = \frac{n^2 (1 - p) [(1 - y^2) + 2yy_1]}{n_1 (1 + y_1^2)}, \quad Y = \frac{2y - y_1 (1 - y^2)}{(1 - y^2) + 2yy_1} \quad \text{for the PP wave.} \quad (8)$$

Let us divide Eq. (7) into imaginary and real parts. Upon corresponding transformations we obtain two equations describing the conditions of reflectionless absorption of the wave in the system in question:

$$Y \sinh 4\pi x y_1 + \sin 4\pi x = 0, \quad (9)$$

$$N (1 + Y^2) = \tanh 2\pi x y_1 - Y \tan 2\pi x. \quad (10)$$

From their simultaneous solution we have

$$\tanh 4\pi xy_1 = \frac{2N}{N^2(1+Y^2)+1}, \quad (11)$$

$$\tan 4\pi x = \frac{2NY}{N^2(1+Y^2)-1}. \quad (12)$$

Since the conditions of reflectionless absorption of the wave in the system are fulfilled at the points of minimum of  $\rho$  as a function of  $Z$ , for coating thicknesses similar to the values which are multiples of  $\lambda_{1d}/4$  we take

$$x = \frac{2N_0 - 1}{4} + \Delta, \quad (13)$$

where  $N_0$  is the number of the zero minimum of  $\rho$  as a function of  $l$  for which  $\rho$  becomes equal to 0 and  $\Delta$  is a small but not zero quantity in the general case; this quantity is determined by simultaneous solution of Eqs. (12) and (13):

$$\Delta = \frac{1}{4\pi} \arctan \frac{2NY}{N^2(1+Y^2)-1}. \quad (14)$$

Substituting expression (13) into Eqs. (11) and (12) and eliminating the quantity  $\Delta$  as the intermediate parameter between them, we obtain

$$\pi(2N_0 - 1) + \arctan \frac{2NY}{N^2(1+Y^2)-1} = \frac{1}{2y_1} \ln \frac{(1+N)^2 + (NY)^2}{(1-N)^2 + (NY)^2}. \quad (15)$$

Equation (15) relates the values of  $n$  and  $y$  and consequently  $\epsilon'$  and  $\epsilon''$  of the substance of the coating of the two-layer system for which the absorption of the incident radiation in the system is total. The required thickness of the coating layer is found, as follows from Eqs. (13) and (14), from the expression

$$\frac{l_0}{\lambda} = \frac{1}{n_1 \sqrt{1-p}} \left[ \frac{(2N_0 - 1)}{4} + \arctan \frac{2NY}{N^2(1+Y^2)-1} \right]. \quad (16)$$

The equations obtained determine occurrence conditions for the reflectionless absorption of electromagnetic radiation in its incidence on the system dielectric–metal at an angle. This total absorption of the wave becomes possible owing to the interference of two waves reflected from the corresponding boundaries of the layered system in question when these waves are opposite in phase but have amplitudes equal in value [4].

Equations (15) and (16) were employed to find the relationships between the selective values of  $\epsilon'$ ,  $\epsilon''$ , and  $l_0$  of the coating substance, the radiation wavelength  $\lambda$ , and the angle of incidence of the wave  $\alpha_0$  for which the conditions of total absorption of electromagnetic radiation in the two-layer system in question are fulfilled. Figure 1 gives as an example the dependences of  $\epsilon''$  on  $\epsilon'$  and of  $l_0/\lambda$  on  $\epsilon'$  at  $N_0 = 1$  respectively for the case of the incidence of transverse and parallel polarized waves on the system. Their behavior as a function of the angle of incidence of the wave depends on the type of its polarization. In the case of the TP wave, the dependences of  $\epsilon''$  on  $\epsilon'$  shift to the abscissa axis with increase in the quantity  $\alpha_0$  and approach it asymptotically at  $\alpha_0 = 90^\circ$ . All the curves of the family lie below the limiting dependence for  $\alpha_0 = 0$ , which corresponds to the case of normal incidence of the wave. The same character of change of the  $\epsilon''(\epsilon')$  curves is preserved in the case of incidence of the PP wave but only in the region of relatively small values of  $\epsilon'$ . Beyond this region, the position of the  $\epsilon''(\epsilon')$  curves changes with increase in  $\alpha_0$ . For high values of  $\epsilon'$  they lie already above the dependence limiting them for  $\alpha_0 = 0$ . For higher-than-average values of  $\alpha_0$  the curves become S-shaped and, as  $\alpha_0$  increases, the point of their inflection shifts toward high values of  $\epsilon'$  with a pronounced increase in the quantity  $\epsilon''$ . Such abnormality in the behavior of the dependences  $\epsilon''(\epsilon')$  is expectable with allowance for the fact that, according to [4], the dependence of the modulus of the reflection coefficient  $\rho$  of the PP

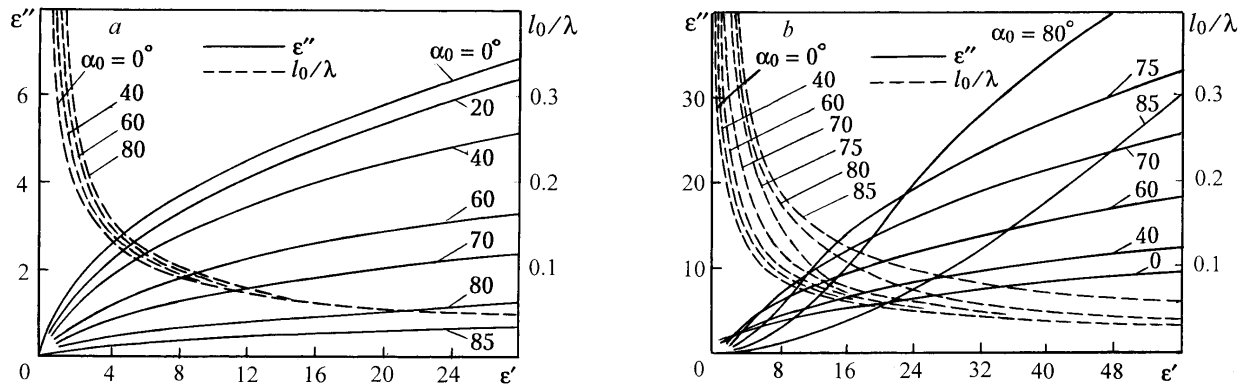


Fig. 1. Relationships between the permittivity  $\epsilon'$ , the dielectric loss  $\epsilon''$ , and the thickness  $l_0$  of the coating layer that correspond to the reflectionless absorption of transverse (a) and parallel (b) polarized waves in the case of their incidence on the two-layer system dielectric–metal at an angle  $\alpha_0$ .

wave on the angle of its incidence on an infinitely thick layer of a nonabsorbing substance with a refractive index  $n$  has a zero maximum lying at a Brewster angle of  $\varphi = \arctan n$ . It is obvious that for  $n > 1$  this well-known effect also manifests itself at  $\varphi > 45^\circ$  in the case of reflection of the wave from a finitely thick layer of an absorbing substance applied to a metal substrate.

Irrespective of the type of polarization of the incident wave, the selective values of the thickness  $l_0$  of the coating layer are always higher than the values which are multiples of  $\lambda_{1d}/4$ . These distinctions grow with increase in  $\alpha_0$  and become significant for the case of incidence of the PP wave.

As the number of the zero minimum  $N_0$  of the function  $\rho(l)$  increases, the families of  $\epsilon''(\epsilon')$  curves (Fig. 1) retain their shape. In the case of incidence of the TP wave, all the curves shift toward the abscissa axis with increase in the quantity  $N_0$ , staying, however, below the dependence limiting them for  $\alpha_0 = 0$ . In the case of incidence of the PP wave, such location of the plots of  $\epsilon''$  as a function of  $\epsilon'$  is retained in a limited range of variation of the values of  $\epsilon'$ .

For the coating substance with values of  $\epsilon'_0$  and  $\epsilon''_0$  which are known for a prescribed frequency of the incident radiation, we can find, graphically or from Eqs. (15) and (16), the selective values of the angle of incidence of the wave and the corresponding thicknesses of the coating layer for which the reflection of the wave is absent. From Fig. 1 it follows that if the operating point with such values of  $\epsilon'_0$  and  $\epsilon''_0$  lies in the coordinate plane  $[\epsilon', \epsilon'']$  at  $N_0 = 1$  above the limiting dependence for  $\alpha_0 = 0$ , the reflectionless absorption of the TP wave in such a coating is impossible. If the operating point with such values of  $\epsilon'_0$  and  $\epsilon''_0$  lies between two limiting dependences with  $N_0 = 1$  and  $N_0 = k$ , for such a coating substance one must observe  $k$  strictly specified angles of incidence of the wave and the corresponding coating-layer thicknesses for which total absorption of the wave occurs. A larger angle of reflectionless incidence of the wave corresponds to a smaller selected thickness of the coating layer. In the case of incidence of the PP wave, its reflectionless absorption in the two-layer system is possible in both variants of position of the operating point in the plane  $[\epsilon', \epsilon'']$ . In both variants, a smaller selective angle of incidence of the wave will correspond to a smaller thickness of the coating layer.

Table 1 gives the selective values (computed based on Eqs. (15) and (16)) of the angles of incidence of the wave and the thicknesses of the coating layer for which conditions for the total absorption of electromagnetic radiation incident on the two-layer system dielectric–metal are created in it. Different polar liquids possessing dispersion in the microwave range were employed in the system as the coating material [5]. For the values of  $\epsilon'$  and  $\epsilon''$  of the liquids indicated in the table and measured at  $\lambda = 3.2$  cm one can expect reflectionless absorption of the waves in these liquids in the range of  $\alpha_0$  and  $l_0$  of  $30\text{--}70^\circ$  and  $0.2\text{--}6$  cm respectively. Unlike the conditions of total absorption of the PP wave in the liquid, the total absorption of the TP wave occurs at the first numbers  $N_0$  of zero minima of the function  $\rho(l)$ . Therefore, the selective values of  $\alpha_0$  of this type of wave will correspond to relatively low but technically realizable values of the thicknesses of the liquids.

Thus, the performed theoretical investigations into the character of the reflection of electromagnetic radiation from an absorbing layer (adjustable for thickness) of a dielectric applied to a metal substrate demonstrate that occur-

TABLE 1. Calculated Values of the Angles of Incidence of the Wave  $\alpha_0$  and the Thickness  $l_0$  of the Coating Layer of the Two-Layer System Dielectric–Metal for Which the Conditions of Reflectionless Absorption of the Incident Radiation Occur. The Temperature of the Substance is  $20^\circ$  and the Incident-Radiation Wavelength is  $\lambda = 3.2$  cm

Coating substance	$\epsilon'$	$\epsilon''$	$N_0$	Transverse polarization of the wave		Parallel polarization of the wave	
				$\alpha_0$ , deg	$l_0/\lambda$	$\alpha_0$ , deg	$l_0/\lambda$
Ethyl alcohol	3.85	1.03	1	61.9	0.464	–	–
			2	–	–	39.5	1.309
			3	–	–	60.1	2.358
			4	–	–	–	–
Propyl alcohol	3.53	1.16	1	56.1	0.483	–	–
			2	–	–	50.9	1.440
			3	–	–	70.9	3.248
			4	–	–	67.3	4.282
			5	–	–	64.7	5.299
Acetone	20.5	3.55	1	51.1	0.179	–	–
			2	–	–	57.6	0.541
			3	–	–	69.6	0.909
			4	–	–	73.9	1.277
			5	–	–	–	–
Anisole	3.04	0.38	1	78.0	0.557	–	–
			2	55.7	1.568	–	–
			3	33.8	2.429	–	–
			4	–	–	31.9	3.382
			5	–	–	50.3	4.624
			6	–	–	55.5	5.773

rence conditions for the phenomenon of total reflectionless absorption of the radiation with a prescribed frequency for the layer thicknesses and the angles of incidence of the wave with a certain polarization which are strictly specified for the coating material employed can exist and experimental observation of this phenomenon is possible.

## NOTATION

$l$ , thickness of the coating layer;  $N_0$ , number of the zero minimum of the reflected wave;  $\alpha_0$ , angle of incidence of the wave on the two-layer system;  $\alpha$ , angle of refraction of the wave at the vacuum–dielectric boundary;  $Z_0$  and  $Z$ , wave resistances of vacuum and of the coating substance;  $\rho$ , modulus of the reflection coefficient of the wave;  $\gamma_0$  and  $\gamma$ , propagation constants of the wave in vacuum and in the coating substance;  $n$ ,  $y$ ,  $\epsilon'$ , and  $\epsilon''$ , refractive index, dielectric loss factor, permittivity, and dielectric loss of the coating substance;  $\lambda$  and  $\lambda_d$ , wavelength in vacuum and in the substance;  $\delta$ , dielectric loss angle;  $n_1$ ,  $y_1$ ,  $\epsilon_1$ ,  $\epsilon_2$ , and  $\delta_1$ , reduced values of the refractive index, the dielectric loss factor, the permittivity, the dielectric loss, and the loss angle of the coating substance;  $\lambda_1$  and  $\lambda_{1d}$ , wavelength in vacuum and in the coating substance respectively in propagation of the wave at a prescribed angle to the plane surfaces bounding it. Subscripts: 0, for the case of reflectionless absorption of electromagnetic radiation; d, for the dielectric coating; in, input.

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